

**BELLCOMM, INC.**

**SUBJECT:** A Potential Nodal Regression Problem  
in the AAP-3/AAP-4 Dual-Rendezvous  
Mission, Case 610

**DATE:** October 30, 1967

**FROM:** W. L. Austin  
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I. Hirsch

**ABSTRACT**

In the AAP-3/AAP-4 dual-rendezvous mission, any delay in the launch of the second vehicle (AAP-4 LM/ATM) will disturb the desired coplanar orientation of the first vehicle (AAP-3 CSM) and the Orbital Workshop (OWS).

Two methods of handling this problem are discussed in this memorandum. On the one hand, the coplanar condition can be restored by an in-orbit plane-change maneuver by the CSM; or, on the other hand, the coplanar condition can be maintained and the plane-change maneuver avoided by raising the CSM altitude to that of the Orbital Workshop for the duration of the AAP-4 launch delay.

For a LM/ATM injection altitude of 240 nautical miles the second alternative mentioned above is clearly preferable, regardless of the length of the launch delay. For a LM/ATM injection altitude of 160 miles some trade-off between the two alternatives warrants further study.

Propellant penalties on the order of 462 lbs of SPS propellant are the minimum in any case. Maximum penalties for long delays can exceed 2000 pounds.

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## MEMORANDUM FOR FILE

### 1. Introduction and Problem Definition

A unique feature of the AAP-3/AAP-4 three-vehicle, dual-rendezvous mission not found in the simpler two-vehicle, single-rendezvous missions is the concurrent "waiting" in orbit of two space vehicles in different orbits anticipating the launch of the third. During the time interval between the launch of the AAP-3 CSM and the AAP-4 LM/ATM (nominally about 1 day) both the CSM and the Orbital Workshop (OWS) orbits are perturbed unequally by the regressional influence of the earth's oblateness. If the launch of the third vehicle (in this case the AAP-4 vehicle) occurs on time, the orbits of the other two vehicles will be in the proper orientation for the subsequent rendezvous operations; that is, the angle between the lines of nodes for the CSM and OWS workshop orbits will be zero. If, however, the AAP-4 launch is delayed, the differences in the regression rates of the orbit planes for the CSM and OWS orbits will generate an increasing nodal angle difference, which will be reflected in the build-up of a wedge angle<sup>1</sup> between the two orbital planes. The delta-V and associated fuel penalty required to realign the lines of nodes in order to remove this wedge angle can be significant, even for a delay of only one day in AAP-4 launching.

In succeeding paragraphs this problem is discussed in detail and the apparent means of factoring it into the overall AAP-3/AAP-4 contingency mission analysis are compared.

It should be noted that this differential nodal regression problem is not the only one associated with a launch delay of the AAP-4 vehicle. Other problems of relative vehicle in-orbit phasing relationships, rendezvous lighting conditions, and abort and recovery considerations must be included in any

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<sup>1</sup>The wedge angle is the small angle of intersection between the CSM and OWS orbital planes.

comprehensive contingency plan for late launches. This memorandum addresses itself only to the nodal regression problem with the presumption that a successful solution, if found, can be factored into an overall contingency plan.

## 2. Details of the Nodal Regression Problem

For orbital inclinations of other than 90 degrees the regression rate of the line of nodes and the corresponding secular perturbation rate for the nodal angle,  $\Omega$ , is a function, among other parameters, of the orbital altitude and inclination of the space vehicle. In the AAP-3/AAP-4 dual-rendezvous mission, which is to be flown at an inclination of 28.86 degrees, the difference in altitude between the 110 nm CSM parking orbit and the 250 nm OWS orbit causes the lower-altitude CSM orbit to regress at a rate approximately 1 degree per day faster than the higher-altitude OWS orbit. As a result of this differential regression, a wedge angle between the CSM and OWS orbital planes develops at the rate of approximately 1/2 degree per day. This increasing wedge angle represents an out-of-plane penalty that must be paid at some time prior to rendezvous. For this analysis, it is assumed the wedge angle would be removed at the intersection of the CSM and OWS orbital planes.

The daily angular difference,  $\Omega_{er}$ , between the lines of nodes of the 110 nm CSM and the 250 nm OWS circular orbits was determined by using the following equations:

$$\Omega_{er} = \left| \frac{86400}{\tau_{nc}} \dot{\Omega}_{lc} - \frac{86400}{\tau_{no}} \dot{\Omega}_{lo} \right| \quad (1)$$

where

- (a)  $\tau_{nc}$  and  $\tau_{no}$  are the perturbed-synodic periods of the CSM and OWS, respectively, in seconds. (earth oblateness perturbations only)
- (b)  $\dot{\Omega}_{lc}$  and  $\dot{\Omega}_{lo}$  are the nodal regression rates per perturbed-synodic period<sup>2</sup> of the CSM and OWS respectively, in radians per second.

$\tau$  and  $\dot{\Omega}$  for circular orbits may be approximated by:

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<sup>2</sup>The synodic period is defined as the time it takes the space vehicle to travel from one ascending node to the next ascending node.

$$\tau \approx 2\pi \sqrt{\frac{r^3}{\mu}} \left\{ 1 - 3J_2 \left( \frac{R}{r} \right)^2 \left( \frac{7 \cos^2 i - 1}{8} \right) \right\} \text{ rad/sec} \quad (2)$$

$$\dot{\Omega} \approx \frac{-3\pi J_2 \cos i}{\left( \frac{r^2}{R} \right)} \text{ rad/rev} \quad (3)$$

where

$r$  = radius of the circular orbit

$i$  = inclination of the vehicle orbit

$\mu$  = gravitational potential of the earth

$J_2$  = coefficient of 2nd term in the series  
expression for the earth's potential function  
when oblateness effects are included

$R$  = mean radius of the earth

The resulting CSM wedge angle,  $\delta_K$ , and wedge angle velocity penalty,  $\Delta V_{\delta_K}$ , for LM/ATM launch delay were determined by using the following equations:

$$\delta_K = 180 - 2 \cos^{-1} \left[ \sin \left( \frac{\Omega_{er}}{2} K \right) \sin i \right] \quad (4)$$

$$\Delta V_{\delta_K} = 2 \sqrt{\frac{\mu}{r}} \left[ \sin \left( \frac{\Omega_{er}}{2} K \right) \sin i \right] \quad (5)$$

where

$K$  = number of days delay in launching the LM/ATM

Figure 1 shows the wedge angle velocity penalty ( $\Delta V_{\delta_K}$ ) and the corresponding propellant ( $W_{p_K}$ ) required as a function of LM/ATM launch delay in days. The fuel penalties for the 160 and 240 nm LM/ATM injection altitudes were determined from the simple rocket equation:

$$W_{p_K} = W_0 \left[ 1 - e^{-\Delta V_{\delta_K} / I_{sp} g_0} \right] \quad (6)$$

where

$W_{p_K}$  = propellant penalty

$W_0$  = initial spacecraft weight

$I_{sp}$  = 312.5 sec. (SPS specific impulse)

### 3. Alternate Solutions

In view of the relatively high fuel penalty associated with removing even small orbital plane orientation differences, consideration has been given to preventing the differential nodal regression rather than realigning the CSM and OWS lines of nodes in the event of a delayed AAP-4 launch. This can be done, in principle, by raising the CSM orbital altitude to that of the OWS when their orbits are co-planar. With both the CSM and OWS at the same altitude for the duration of the AAP-4 launch delay, the nodal regression rates will be the same and consequently, the CSM and OWS lines of nodes will remain in the same plane. In addition, equi-altitude orbits will allow the CSM and OWS to maintain essentially the same phase relationship during the launch delay period, thus helping to alleviate a serious multi-orbit phasing problem that could also develop.

Of course, varying the CSM orbital altitude in this manner in itself incurs a fuel penalty; and hence, a trade-off situation develops between realigning the lines of nodes on the one hand and preventing the differential nodal regression

on the other. Figures 2 and 3 show schematically the changes in the CSM mission required to prevent the nodal regression problem. For the baseline mission in which the LM/ATM is inserted at 240 nm altitude, the retrieval penalty associated with the CSM's return from 250 to 230 nm for rendezvous is relatively small (482 lbs propellant). In the case of the 160 nm LM/ATM orbit that has been proposed, the fuel required to retrieve the LM/ATM from this lower altitude is significantly larger (1403 lbs propellant).

The delta-V's and associated propellant expenditures shown in Figures 2 and 3 for the pre-terminal phase maneuvers, were computed from the basic rocket equation<sup>3</sup> for a Hohmann transfer between two circular orbits. The values for the terminal phase maneuvers (which in all cases were initiated from 10 nm below the target vehicle) were obtained from data in references 1 and 2. It should be recognized that the delta-V and corresponding propellant values used here reflect an ideal minimum-fuel situation. In the course of detailed mission planning a more complete delta-V and propellant budget, including among other factors allowances for rendezvous and docking dispersions, RCS ullage burns, and gauging errors must be developed. However, for comparative purposes in analyzing alternate schemes the simplified computations used here should be satisfactory. A CSM insertion weight of 36,000 lbs in the 110 nm circular orbit, and a LM/ATM insertion weight of 24,800 lbs in the 240 nm circular orbit (29,500 lbs when the LM/ATM is inserted into a 160 nm circular orbit) are based on the MSC data used in their discussions at the AAP Guidance, Performance, and Dynamics (GP&D) Subpanel meeting on August 24, 1967 at MSFC. The OWS was assumed to be in a 250 nm circular orbit, having decayed approximately 10 nm from the original 260 nm circular orbit in which it was left at the completion of the AAP 1/AAP 2 dual-rendezvous mission.

For comparison, the additional fuel requirements for performing the regression-prevention maneuvers of Figures 2 and 3 have been plotted along with the regression-correction data of Figure 1. These combined data and the resulting "break-even" points for both LM/ATM orbits are shown in Figure 4. Note that for the 240 nm LM/ATM orbit, even a very slight delay in AAP-4 launch makes the placing of the CSM at OWS altitude an attractive alternative. For the 160 nm LM/ATM injection case, the break-even point occurs somewhat later. An important consideration in both cases, is the fact that the penalty associated with raising the CSM altitude is a pre-determined propellant quantity and mission maneuver that can

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<sup>3</sup>Equation (6) on p. 4

be factored into contingency mission planning. In contrast to this, the penalty to realign the CSM and OWS lines of nodes is not a constant, but continues to increase with the AAP-4 delay time and is correspondingly more difficult to account for in mission planning.

#### 4. Evaluation of Solution Strategies

Based on the considerations discussed in the preceding paragraphs, several strategies for obtaining an optimum solution to the nodal regression problem suggest themselves. For the nominal mission with the LM/ATM injection altitude at 240 nm the fuel-penalty break-even point between the two alternative solutions occurs after a delay of less than 24 hours (see Figure 2). Thus, for this situation, any postponement of the AAP-4 launch would be best handled by raising the CSM to the OWS altitude until the LM/ATM launch is successfully achieved. For the possible alternative mission in which the LM/ATM is injected at only 160 nm, (see Figure 3) the break-even point occurs after a 3-day launch delay. Therefore some consideration must be given to correcting rather than preventing the nodal regression problem. Based on discussions at the 3rd and 4th GP&D subpanel meetings, current estimates for the probability of successfully launching the second vehicle of a dual launch mission on time are discouragingly small--on the order of a 20% or .2 probability. In addition, any problem causing a slip in the AAP-4 launch will lead to a mean expected delay of about 48 hours resulting in an expected velocity penalty,  $E(\Delta V_0)$ , of 428 fps and an expected fuel penalty,  $E(W_p)$ , of 1503 lbs. See Figure 4. With this in mind, a check of Figure 1 shows that by launching the CSM with a built in  $-1.0^\circ$  misalignment of the CSM and OWS lines of nodes, a 48-hour delay could be accommodated with a fuel penalty equal to that required for a 24-hour delay. In the event of an on-time launch of the AAP-4 vehicle, a plane change maneuver would be required; or an in-orbit wait of 24 hours would be required to allow for the removal of the built in  $-1$  degree difference in CSM-OWS lines of nodes by normal regression. This latter alternative may be procedurally unacceptable from a mission planning standpoint. Nevertheless, for the 160 nm LM/ATM insertion altitude, some consideration should be given to realigning the CSM-OWS lines of nodes rather than placing the CSM in an equi-period orbit with the OWS. If the LM/ATM launch delay can be guaranteed not to exceed 72 hours, the relatively high penalty associated with a CSM retrieval maneuver from OWS altitude down to 160 nm and back to the OWS again may make the corrective burn a better choice.

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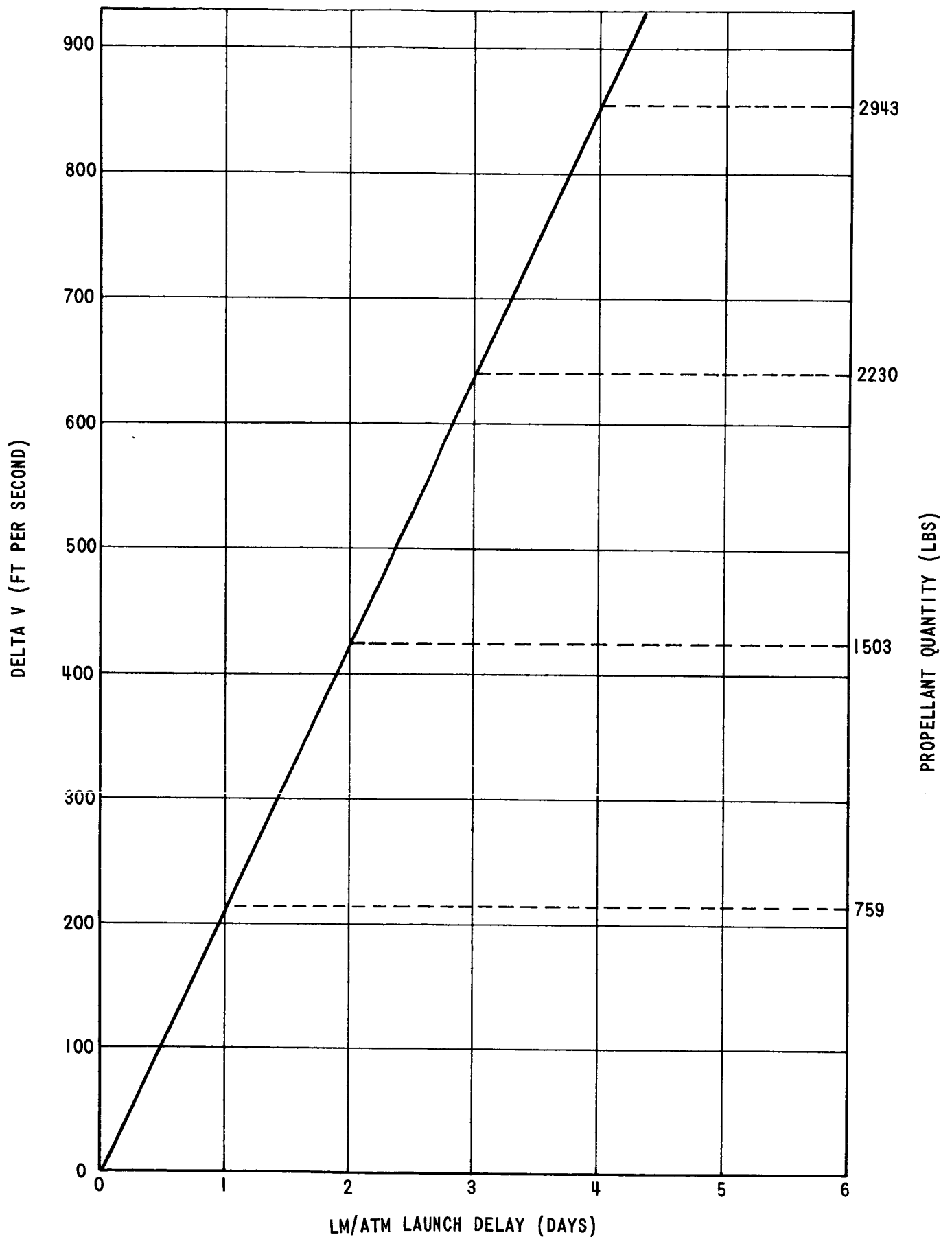


FIGURE 1 - NODAL REGRESSION  $\Delta V$  AND SPS PROPELLANT PENALTY vs. LM/ATM LAUNCH DELAY



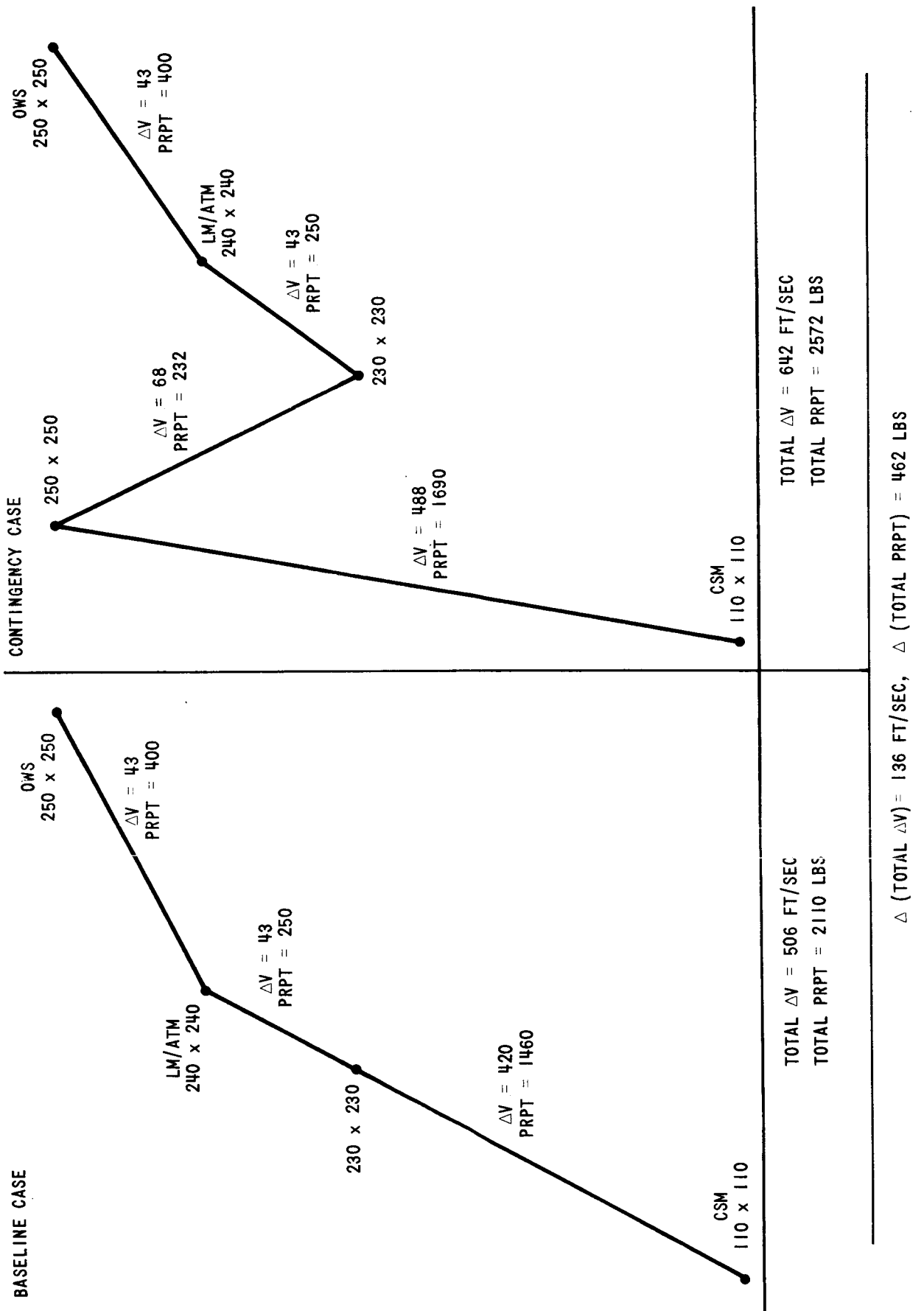


FIGURE 2 -  $\Delta V$  AND PROPELLANT REQUIREMENTS FOR BASELINE AND CONTINGENCY CASES WITH LM/ATM AT 240 N.M.I.

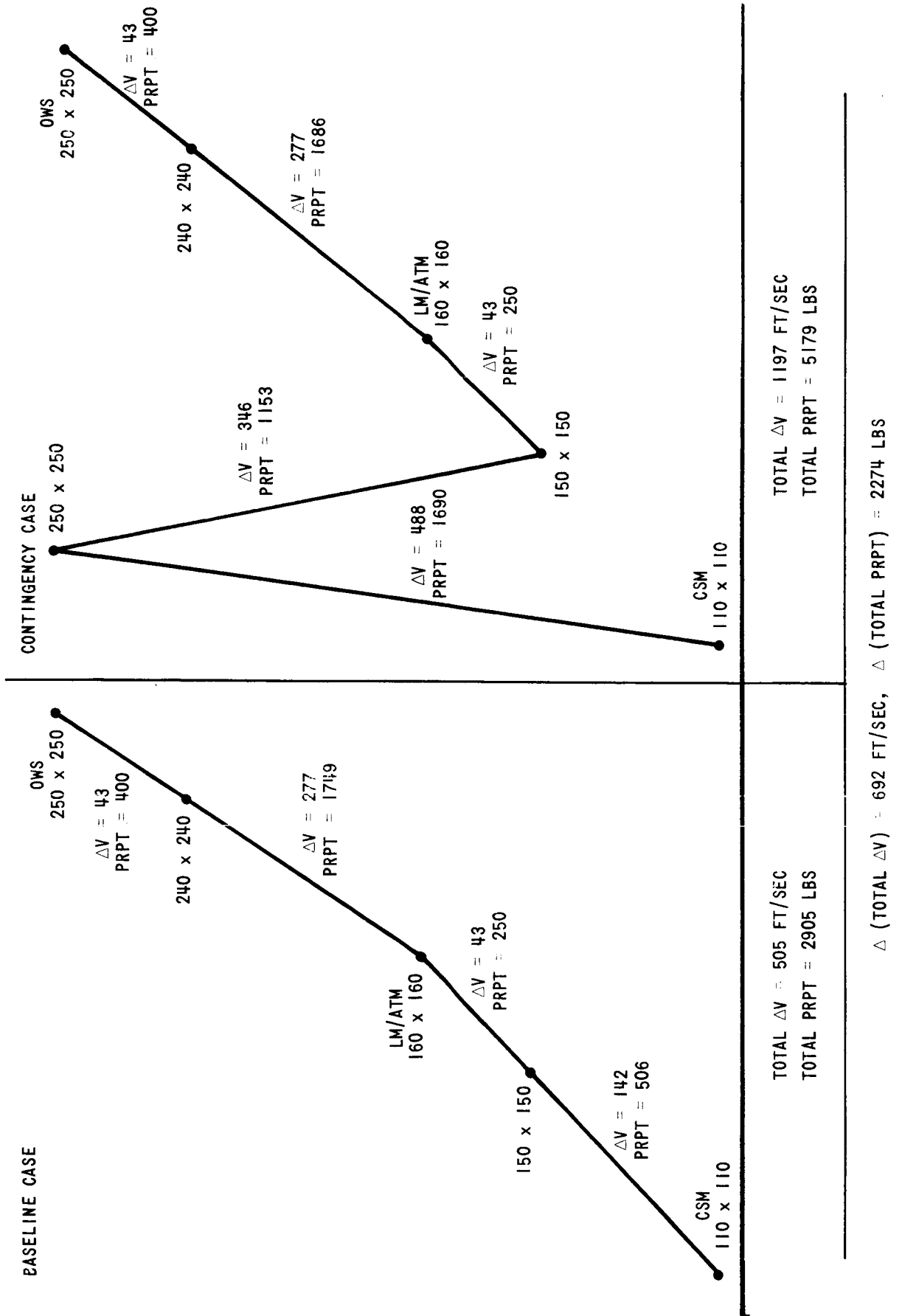


FIGURE 3 -  $\Delta V$  AND PROPELLANT REQUIREMENTS FOR BASELINE AND CONTINGENCY CASES WITH LM/ATM AT 160 N.M.I.

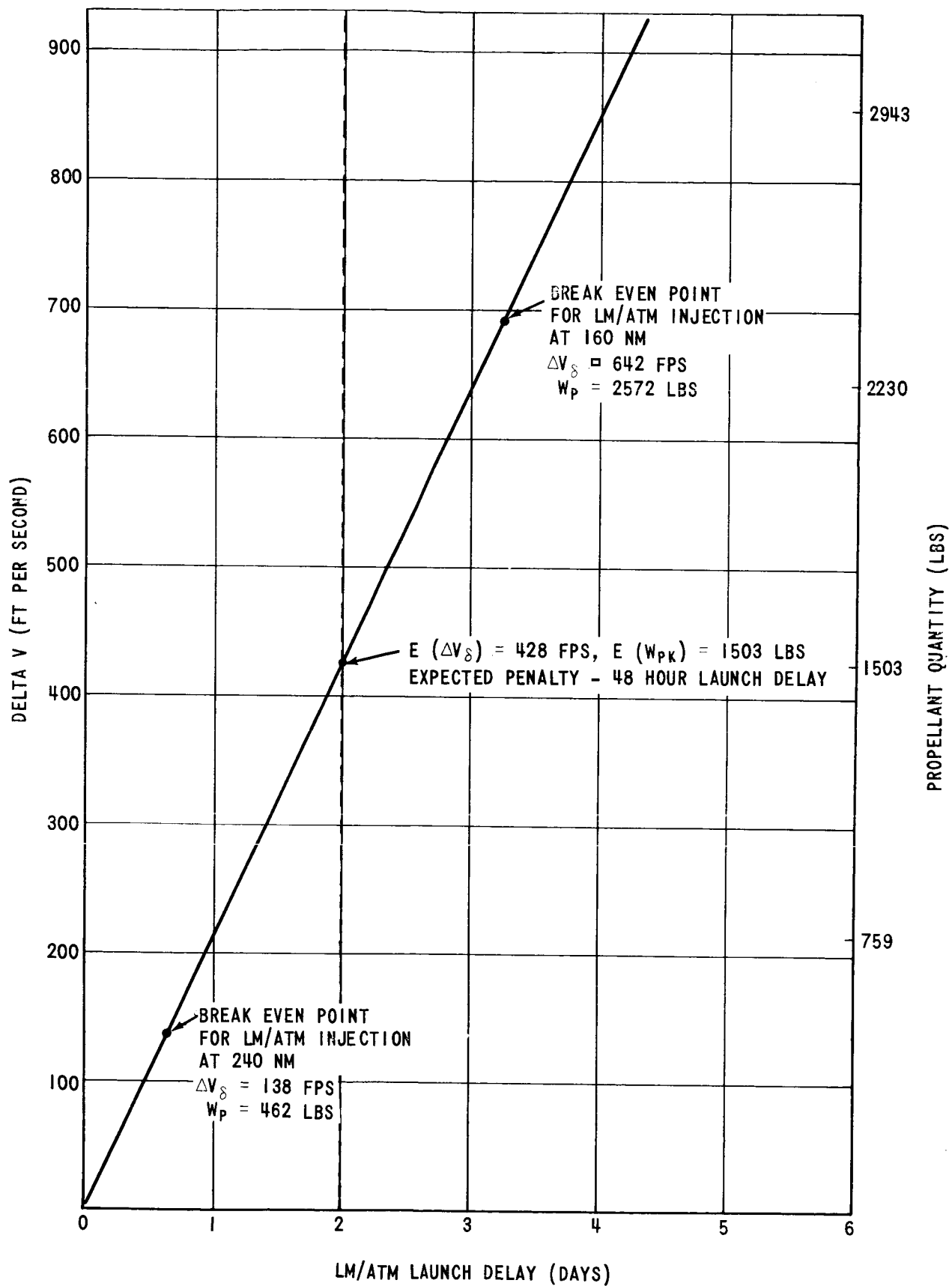


FIGURE 4 - BREAK EVEN POINTS vs. LM/ATM LAUNCH DELAY

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